Engineering Notes

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Least Acceleration Motion for Given Terminal Conditions

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MOTION of a particle from a given initial position P_i to a given final position P_f with given terminal velocities v_i and v_f and given transfer time $t_f - t_i$ may be performed in infinitely many ways. How do we choose the motion requiring the least upper limit of acceleration magnitude needed?†

In Fig. 1 the initial velocity vector $\boldsymbol{v}_i$, the final velocity vector $\boldsymbol{v}_f$, and the mean velocity vector $\boldsymbol{v}_m$, all originating at P_i, define a fixed plane π_0. Another plane π (not shown) parallel to π_0 contains the terminus of the vector $(t - t_i)\boldsymbol{v}_m$. Moving with the constant mean velocity $\boldsymbol{v}_m$, the plane π defines an inertial two-dimensional coordinate system within which the optimal particle motion takes place, simply because additional motion normal to the plane π would increase the acceleration magnitude. Our problem is, hence, reduced from three- to two-dimensional. Choosing the origin at the terminus of the vector $(t - t_i)\boldsymbol{v}_m$, we have a particle motion in the plane π starting from the origin with the velocity $\boldsymbol{v}_i - \boldsymbol{v}_m$ and returning to the origin with the velocity $\boldsymbol{v}_f - \boldsymbol{v}_m$. Hence it follows that

$$\int_{t_i}^{t_f} (\boldsymbol{v} - \boldsymbol{v}_m)\, \mathrm{d}t = 0$$

i.e., the point corresponding to the "center of mass" of the hodograph must be situated at the origin, dt now playing the role of mass element of the hodograph curve.

The case of even mass distribution now naturally recommends itself, and leads us to consider the case when the acceleration magnitude is constant during the motion. It follows easily that a catenary shaped hogograph then becomes optimal. This is due to the fact that the center of mass of a suspended chain of a certain length is lower for the catenary equilibrium configuration than for any other configuration. Hence the "shortest" hodograph curve in question is catenary shaped, and the acceleration magnitude being equal to "curve length" divided by transfer time is minimized.

Further use of the chain analogy shows that a nonconstant magnitude of acceleration may be excluded as the optimal possibility. Considering the fact that a chain of given mass with uneven mass distribution lowers its center of mass if its thicker parts are stretched out to reduce mass distribution, this becomes more or less evident.

The above considerations for motion relative to an inertial frame are easily generalized to the case of a constant gravitational field, as, e.g., when considering the maneuver of a combat aircraft. To compensate for gravitation, we simply move the final position to a point $\frac{1}{2}g(t_f - t_i)^2$ above P_f and change the final velocity from $\boldsymbol{v}_f$ to $\boldsymbol{v}_f - \boldsymbol{g}(t_f - t_i)$.

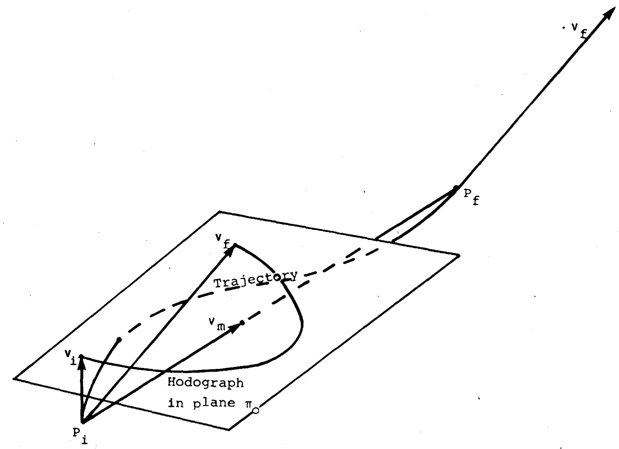

Fig. 1 Least accelerated motion with given terminals.

Received Oct. 6, 1983; revision received Dec. 19, 1983.

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†This problem was raised and solved in a particular case by Harald B. Klepp of the Royal Norwegian Naval Academy. The solution in the general case was conjectured by Mari Vårhus of the University of Trondheim, Norway.

Separation of Time Scales in Aircraft Trajectory Optimization

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Introduction

SINGULAR perturbation methods have been shown by many authors[1-19] to be effective and efficient in the analysis and computation of aircraft optimal trajectories because they reduce the order of individual integrations. Singular perturbation methods depend upon identifying the characteristic "small" parameters of the dynamical systems or, equivalently, upon identifying the characteristic time scales of the state variables. Several authors[4,15] have observed that there is presently no method of casting complex (high-order, highly coupled, highly nonlinear) aircraft trajectory optimization problems in a singular perturbation form which is both rigorous and practical, and that this constitutes a gap

Presented as Paper 83-2136 at the AIAA Atmospheric Flight Mechanics Conference, Gatlinburg, Tenn., Aug. 15-17, 1983; received Nov, 21, 1983; revision received Jan. 11, 1984. This paper is declared a work of the U.S. Government and therefore is in the public domain.

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